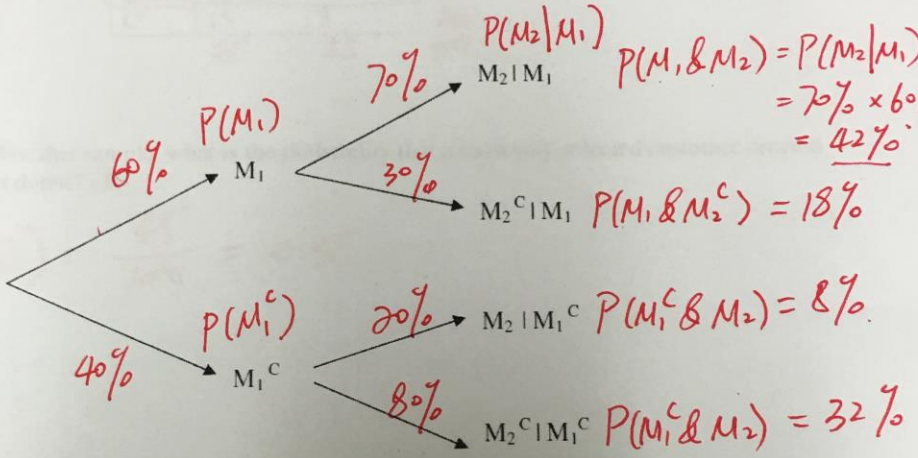
1. Suppose a basketball player with two attempts at a free throw has a 60% chance of making his first shot. If he makes the first shot, there is a 70% chance that he will make his second shot. If he misses his first shot, there is a 20% chance the he will make his second shot.

Let M_1 = he makes his first shot
Let M_1^c = he misses his first shot
Let M_2 = he makes his second shot
Let M_2^c = he misses his second shot



$P(M_1 \& M_2) = P(M_2|M_1) \cdot P(M_1)$
$= 70\% \times 60\%$
$= 42\%$

$P(M_1 \& M_2^c) = 18\%$

$P(M_1^c \& M_2) = 8\%$

$P(M_1^c \& M_2) = 32\%$

a. Complete the tree diagram above by adding 'weights' to the branches. (6)

b. What is the probability that he misses both shots? (4)

	M_2	M_2^c	
M_1	42%	18%	60%
M_1^c	8%	32%	40%
	50%	50%	100%

$P(M_1^c \text{ and } M_2^c) = 32\%$

$P(M_2|M_1) = 70\%$
$P(M_2|M_1^c) = 20\%$

1. $P(D|C) = P(D)$
2. $P(C|D) = P(C)$
3. $P(D \& C) = P(D) \times P(C)$

2. Suppose a restaurant owner wants to study the relationship between coffee and donut sales at his diner. He records orders for 100 breakfast customers. The results are below.

Let **D** = a person orders donuts
Let $\mathbf{D^c}$ = a person does not order donuts
Let **C** = a person orders coffee
Let $\mathbf{C^c}$ = a person does not order coffee

	D	D^c	
C	30	50	80
C^c	15	5	20
	45	55	100

a. For this sample, what is the probability that a randomly selected customer ordered a donut? (5)

$$P(D) = \frac{45}{100} = 0.45$$

b. For this sample, given that someone ordered coffee, what is the probability that they also ordered a donut? (5)

$$P(D|C) = \frac{30}{80} = 0.375$$

$$P(D|C) = \frac{P(D \& C)}{P(C)} = \frac{\frac{30}{100}}{\frac{80}{100}} = \frac{0.3}{0.8} = 0.375$$

c. For this sample, are D and C independent events? Justify your answer using probabilities. (You will receive zero credit for your answer without justification.) (5)

$P(D) = 0.45$
$P(D|C) = 0.375$

$P(D) \neq P(D|C)$

so, not independent.

3. The table below shows the probability distribution for the number of body piercings for STAT 201 students (based on a survey of STAT 201 students in Fall 2010).

x	P(x)	$x \cdot P(x)$
0	0.30	0
1	0.09	0.09
2	0.20	0.4
3	0.11	0.33
4	0.11	0.44
5	0.09	0.45
6	0.05	0.3
9	0.02	0.18
10	0.03	0.3

a. Verify that this is a valid probability distribution. (5)

1. 9 different x's, so it is countable.
2. For all x, $0 \leq P(x) \leq 1$
3. $\Sigma P(x) = 0.3 + 0.09 + 0.2 + \cdots + 0.03 = 1$

This is a valid probability distribution

c. What is the average number of body piercings for STAT 201 students? (5)

$$\text{mean} = \Sigma x P(x)$$
$$= 0 + 0.09 + 0.4 + 0.33 + 0.44 + 0.45 + 0.3 + 0.18 + 0.3$$
$$= 2.49$$

$\mu = 320$, $\sigma = 50$

4. According to Dump and Run, Inc, the mean amount of paper thrown away by college students is 320 pounds per year. (Source: http://www.dumpandrun.org/garbage.htm) Assume this is a bell-shaped distribution with a standard deviation of 50 pounds.

a. What proportion of college students throw away less than 200 pounds of paper per year? (5)

$P(X<200)$

$z_{200} = \frac{200-320}{50} = -2.4$ $\quad P(z<-2.4) = 0.0082$

$P(X<200) = P(z<-2.4) = 0.0082$

b. What proportion of college students throw away between 300 and 400 pounds of paper per year? (5)

$P(x<300) = P(z < \frac{300-320}{50}) = P(z<-0.4) = 0.3446$

$P(x<400) = P(z < \frac{400-320}{50}) = P(z<1.6) = 0.9452$

$P(300 < x < 400) = 0.9452 - 0.3446 = 0.6006$

$P(X>300) = 1 - P(x<300) = 1 - 0.3446$

c. What amount of paper thrown away is at the 25th percentile? ? (5)

$P(z<-0.67) = 0.25$ $\quad x = \mu + \sigma \cdot z = 320 + 50 \times (-0.67) = 286.5$

5. In a particular class, 96% of students are right-handed. If 5 of these students are picked randomly, what is the probability that exactly 3 are right handed? (5)

$n = 5$, $p = 0.96$; $q = 1 - p = 1 - 0.96 = 0.04$

$P(x=3) = {}_5C_3 \times p^3 \times q^{5-3}$

$5! = 5 \times 4 \times 3 \times 2 \times 1$

$3! = 3 \times 2 \times 1$

$2! = 2 \times 1$

$= \frac{5!}{3!(5-3)!} \times 0.96^3 \times 0.04^2$

$= \frac{5 \times 4 \times 3 \times 2 \times 1}{3 \times 2 \times 1 \times 2 \times 1} \times 0.96^3 \times 0.04^2$

$= \frac{5 \times 4}{2} \times 0.96^3 \times 0.04^2$

$= 0.014$

$n = 150$, $p = 0.2$, $q = 0.8$

6. To promote business, a local ice cream parlor mails out 150 coupons to receive a free ice cream cone on National Ice Cream Day to a random selection of local addresses. They know from past experience that when a customer receives a coupon, there is a 20% chance that the customer will redeem it (bring it in and use it.) Let X = the number of coupons that are redeemed.

a. Find the mean of X. (5)

$\text{mean} = n \times p = 150 \times 0.2 = 30$

b. Find the standard deviation of X. (5)

$\text{std.dev} = \sqrt{n \times p \times q} = \sqrt{150 \times 0.2 \times 0.8} = 4.9$

c. Would it be surprising if only 10 customers redeemed the coupon? Use the Emprirical Rule to justify your answer. (5)

$n = 150$, $p = 0.2$, $q = 0.8$

$n \times p = 150 \times 0.2 = 30 \geq 15$

$n \times q = 150 \times 0.8 = 120 \geq 15$

$$z_{10} = \frac{10 - 30}{4.9} \approx -4 < -3$$

so by the Emprirical Rule 10 customers redeemed the coupon is an outlier.